INVESTMENT RETURNS AND INFLATION MODELS: SOME AUSTRALIAN EVIDENCE

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ABSTRACT

The development of stochastic investment models for actuarial and investment applications has become an important area of interest to actuaries. This paper reports the application of some techniques of modern time series and econometric analysis to Australian inflation, share market and interest rate data. It considers unit roots, cointegration and state space models. Some of the results from this analysis are not reflected in published stochastic investment models.

KEYWORDS

Stochastic models; Unit roots; Cointegration; State space models; Inflation; Interest rates; Share market

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1. INTRODUCTION

1.1 The main aim of this paper is to apply some of the techniques of modern time series and econometric analysis to analyse investment data in order to better understand the nature of long run relationships in investment series typically used in stochastic investment models. These relationships are fundamental to the structure of any model to be used in actuarial applications. The analysis is based on Australian data. For other stochastic investment modelling studies for actuarial applications that have investigated such relationships refer to Wilkie (1986, 1995) and Thomson (1996).

1.2 The paper attempts to identify and address fundamental issues that need to be considered before developing a particular stochastic investment model. It has identified many structural features of investment models that should be included in such a model. Many of these features are not found in published stochastic investment models for actuarial applications. The paper does not present a stochastic investment model. The detail required for a stochastic investment model will depend on the application.

1.3 Transfer functions were used to fit models to Australian investment data. These were found not to be appropriate for investment modelling analysis for the Australian asset returns series data since there was evidence of feedback relationships between many of the series. State space models were then fitted to the asset returns data and the inflation series allowing for feedback between inflation and asset class returns. The fitted models are reported in this paper.

1.4 The analysis in this paper uses quarterly data. Stochastic investment models are used in practice to establish strategic asset allocations and to examine solvency and capital adequacy. Long run asset allocation strategies are often determined using an annual model on the assumption that cash flows occur at annual intervals. In practice this will be a crude approximation to the timing of cash flows and a higher frequency model will be preferred. Similarly the capital requirements for meeting a solvency test will be much less stringent when solvency is tested at annual time intervals than at quarterly intervals.

1.5 There are many important issues in stochastic investment modelling that require further investigation. These include modelling structural changes that have occurred in the economy using regime-switching models (Harris, 1997, Garcia and Perron, 1996 and van Norden and Vigfusson, 1996) and allowing for other time varying components of the series such as heteroscedasticity. Parameter estimation and stability of parameters require further investigation. Parameter and model uncertainty also needs to be incorporated.

1.6 Structural time series models provide a framework for analysing and developing stochastic investment models. Such models can be developed using a state space formulation. Such an approach provides a number of significant advantages over more dated time series techniques. These are:

a) Models reflect the important structural characteristics of the data.
b) Model parameters are readily interpretable.
c) Feedback mechanisms (bi-directional causality) are included.
d) Stationarity does not have to be assumed.
e) On-line model maintenance and updating on receipt of additional information.

Further research is required in this area. It is hoped that this paper will provide a foundation for that research.

2. DATA

2.1 As far as possible, the structure of a model should be consistent with validated or widely accepted economic and financial theory. Often the theory will rely on empirical data for justification and the results from testing the theory will be inconclusive. A statistical analysis of historical data will also provide useful insights into the features of past experience that the model will need to capture. The model structure should be consistent with historical data. Parameter estimation will usually be based on historical data. Since the model will be used for projection into the future a greater weight may be given to the most recent data.

2.2 The data used for the empirical analysis in this research were taken from the Reserve Bank of Australia Bulletin database. The study uses quarterly data in contrast to most other studies in this area that use annual data. The reasons for using quarterly data, rather than monthly or some other higher frequency, is that this is the highest frequency for which many of the main economic and investment series are available. It is also a frequency that is suited to most practical applications. A quarterly model will allow the results of financial analysis and projections to be reviewed on a more frequent basis.

2.3 Different series are available over different time periods. The longest time period for which data were available on a quarterly basis for all of the financial and economic series was from September 1969. Individual series were available for differing time periods. The series considered were Consumer Price Index - All groups (CPI), the All Ordinaries Share Price Index (SPI), Average Weekly Earnings - adult males (AWE), share dividend yields, 90 day bank bill yields, 2 year Treasury bond yields, 5 year Treasury bond yields, and 10 year Treasury bond yields. An index of dividends was constructed from the dividend yield and the Share Price Index series. Logarithms and differences of the logarithms are used in the analysis of the CPI, SPI, AWE and dividends. The differences in the logarithms of the level of a series are the continuously compounded equivalent growth rates of the series.

2.4 Appendix A sets out summary statistics for the series used. It is important to note that a number of the series can not be assumed to be independent and identically distributed normal variables. Although the data are not actual investment returns, since they are interest rates and indices, they do suggest that the normal distribution assumptions of mean-variance models often used in determining optimal asset allocation strategies should at least be examined before using such models.

3. UNIT ROOTS AND STATIONARY SERIES

3.1 Many of the series used in stochastic investment modelling are non-stationary. That is their mean, variance and auto-covariances may depend on time. For example the level of the Consumer Price Index, the level of the Share Price Index and the level of a dividend index can be seen to be non-stationary by simple inspection of a time series plot. It is less clear whether or not interest rates have stationary distributions and this cannot easily be determined by inspection of a time series plot. Rates of changes in indices or rates of return have been used in stochastic investment models and this might be justified because they can be considered as “natural” variables to use. In the case where the variance depends on time then the use of a logarithmic transform may result in a stationary series.

3.2 The level of the Consumer Price Index, the level of the Share Price Index and the level of a dividend index are non-stationary. Their expected value clearly depends on time. This has lead researchers to difference the data in order to obtain a stationary series for modelling. Wilkie (1986) and Carter (1991) used differenced data, as does Harris (1994, 1995) for equity and inflation series. In Carter (1991) the order of differencing was decided using more traditional time series techniques based on the sample autocorrelations. FitzHerbert (1992) fits a deterministic trend to various index levels instead of taking differences. Neither FitzHerbert (1992) nor Harris (1994, 1995) conduct formal tests for stationarity of the series used in their models.

3.3 If the level of a series is non-stationary but the difference of the series is stationary then the series is said to contain a “unit root”, be “integrated or order 1”, or be “difference stationary”. The following outline of the concept of a unit root is based on Holden and Perman (1994). Consider the first order autoregressive process

$$x_t = \rho x_{t-1} + \varepsilon_t \,, \; t = ..., -1, 0, 1, ...$$

where ε_t is a sequence of independent and identically distributed random variables with mean zero and variance σ^2 and $|\rho|<1$. Define the lag operator L as $Lx_t=x_{t-1}$ and write

$$x_t - \rho x_{t-1} = x_t - \rho L x_t = (1-\rho L)x_t$$

so that

$$\begin{aligned} x_t &= (1-\rho L)^{-1}\varepsilon_t \\ &= (1+\rho L+\rho^2 L^2+....)\varepsilon_t \\ &= \varepsilon_t+\rho\varepsilon_{t-1}+\rho^2\varepsilon_{t-2}+... \end{aligned}$$

3.4 It follows that

$$E(x_t)=0$$

$$\text{var}(x_t)=\frac{\sigma^2}{(1-\rho^2)}$$

$$\text{cov}(x_t, x_{t-\tau})=\frac{\rho^\tau\sigma^2}{(1-\rho^2)}, \tau=1,2,....$$

3.5 Since the expected value, variance and autocovariance do not depend on time the process is stationary provided $|\rho|<1$. Note that we can write $f(L)x_t=\varepsilon_t$ where $f(L)=1-\rho L$ is a linear function of L with root given by $L=1/\rho$.

3.6 The function f(L) has a unit root when $\rho=1$. In this case, assuming the process starts at time t=0, we have

$$\begin{aligned} x_t &= x_{t-1}+\varepsilon_t, \; t=0,1,2.... \\ &= x_0+\varepsilon_t+\varepsilon_{t-1}+\ldots+\varepsilon_1 \end{aligned}$$

so that

$$E(x_t)=x_0$$

$$\text{var}(x_t)=t\sigma^2$$

$$\text{cor}(x_t, x_{t-\tau})=\sqrt{\frac{t-\tau}{t}}, \tau=1,2,....$$

and the process is no longer stationary. However, if we take differences of the series then these will be stationary. Hence the process is said to be “difference stationary”.

3.7 It is important to understand that the existence of unit roots determines the nature of the trends in the series. If a series contains a unit root then the trend in the series is stochastic and shocks to the series will be permanent. In the case above with $\rho=1$ the level of the series is an accumulation of past random shocks. If the series does not contain a unit root then the series is “trend stationary”. The trend in the series will be deterministic and shocks to the series will be transitory. This has major implications for investment models in actuarial applications.

3.8 The other aspect of unit roots is that if they exist in a series and differences are not used in model fitting and parameter estimation then the statistical properties of the parameter estimates for the model will not be standard. The use of standard results for model identification and parameter estimation can result in an incorrect model structure and unreliable parameter estimates. Insignificant parameters are more likely to be accepted as being significant. These issues are discussed in Holden and Perman (1994).

3.9 These are all significant reasons that make testing for unit roots in a series for use in developing an investment model critical. Formal statistical tests for unit roots have been developed over the past decade in the econometric literature. Unit root tests are described in many articles and books including Dickey and Fuller (1979) and Mills (1993). These procedures are implemented in various statistical and econometric packages such as SHAZAM (1993).

3.10 In order to test for unit roots in the series x_t the following regressions are fitted:

$$x_t = \alpha + \rho x_{t-1} + \varepsilon_t \tag{1}$$

and

$$x_t = \alpha + \beta t + \rho x_{t-1} + \varepsilon_t \tag{2}$$

where ε_t are assumed to be independent and identically distributed.

3.11 These regressions are often written in the equivalent form

$$\Delta x_t = x_t - x_{t-1} = \alpha + (\rho - 1) x_{t-1} + \varepsilon_t$$
$$\Delta x_t = x_t - x_{t-1} = \alpha + \beta t + (\rho - 1) x_{t-1} + \varepsilon_t$$

in which case they are referred to as Dickey-Fuller regressions (Dickey and Fuller, 1979, 1981).

3.12 If the value of ρ is equal to one and α is non-zero then x_t is integrated or order 1 and (1) defines x_t as a random walk around a linear time trend and (2) defines x_t as a random walk around a non-linear (quadratic) time trend. This can be seen by substituting ρ= 1 and rearranging to get:

$$x_t^* = x_{t-1}^* + \varepsilon_t$$

with

$$x_t^* = x_t - \alpha t$$

for (1), and

$$x_t^* = x_t - \left[\alpha + \frac{\beta}{2}\right] t - \frac{\beta}{2} t^2$$

for (2).

3.13 If the ε_t are not i.i.d. then the following regressions, referred to as augmented Dickey-Fuller regressions, are used:

$$\Delta x_t = \alpha_0 + \alpha_1 x_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta x_{t-1} + \varepsilon_t \tag{3}$$

$$\Delta x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 t + \sum_{j=1}^{p} \gamma_j \Delta x_{t-1} + \varepsilon_t \tag{4}$$

where p is selected to ensure the errors are uncorrelated. Note that α_1 = 1-ρ. These will be the regressions used in this paper to examine Australian data.

3.14 The procedure for testing for unit roots and determining the order of integration uses the t-statistic of the coefficient α_1 of x_{t-1} in the regressions given by (3) and (4). The null hypothesis is that the series is non-stationary with α_1 = 0 (i.e. ρ = 1) and the alternative is that

$\alpha_1 < 0$. If the null is rejected then this is evidence that x_t is a stationary series. If the null is not rejected then differences of the series are taken and the differences tested for a unit root. When the null is eventually rejected the level of differencing required to reject the null determines the order of integration of the series. Usually this requires at most one order of differencing for financial and economic series.

3.15 The critical values used for testing for a unit root depend on the assumed underlying data generating model for the null hypothesis. If both α_0 and α_2 are zero then the t-statistic under the null hypothesis for the regressions in (3) and (4) has a non-standard distribution (Dickey and Fuller, 1979) and is compared with the table of critical values found in Fuller (1976, p. 373). If α_0 is non-zero and α_2 is zero then the test statistic for regression (4) is non-standard but is standard normal for regression (3). If both α_0 and α_2 are non-zero then the limiting distribution for the test statistics in both regression (3) and (4) is standard normal.

3.16 Dickey and Fuller (1981) provide critical values for a range of F test statistics based on residual sums of squares for

ϕ_1 using Equation (3) with Null $\alpha_0=0, \alpha_1=0$

ϕ_2 using Equation (4) with Null $\alpha_0=0, \alpha_2=0, \alpha_1=0$

ϕ_3 using Equation (4) with Null $\alpha_0 \neq 0, \alpha_2=0, \alpha_1=0$

3.17 Given the above, it is necessary to use a sequential procedure to determine critical values for testing for unit roots. Holden and Perman (1994) suggest such a procedure for testing for unit roots. The following is a summary of the main steps in this procedure:

Step 1: Estimate the regression (4).

Step 2: Use ϕ_3 and critical values from Dickey and Fuller (1981) (with $\alpha_0 \neq 0$) to test the null hypothesis $\alpha_2=0, \alpha_1=0$ against the alternative hypothesis $\alpha_2 \neq 0$ or $\alpha_1 \neq 0$. If the null is not rejected then go to Step 5.

Step 3: Now it is necessary to determine if $\alpha_2 \neq 0, \alpha_1=0$, or $\alpha_2=0, \alpha_1 \neq 0$ or $\alpha_2 \neq 0, \alpha_1 \neq 0$. Test for $\alpha_1=0$ using the t statistic from step 1 and standard normal tables. If this null is not rejected then conclude that $\alpha_2 \neq 0, \alpha_1=0$ and the series has a unit root and non-linear trend and stop. If the null is rejected then proceed to the next step.

Step 4: The null is rejected in Step 3. There is no unit root and the series is stationary. A conventional t-test for $\alpha_2=0$ is used to test for a trend. If this null is rejected then the series is stationary around a linear trend and the process stops. A conventional t-test is used to test for a constant $\alpha_0 \neq 0$.

Step 5: If in Step 2 the null is not rejected then the series has a unit root with no trend and possibly with drift. The unit root can be confirmed using the non-standard critical values for the null $\alpha_1=0$. Non-zero drift is tested for in step 6.

Step 6: To test for non-zero drift use ϕ_2 and the Dickey and Fuller (1981) critical values. If the null is not rejected then the evidence suggests that the series is a random walk without drift. If the null is rejected then the series is a random walk with drift. Finally proceed to step 7.

Step 7: Regression (3) is used and ϕ_1 used to test the null α_0=0, α_1=0. This will confirm the results of earlier steps.

3.18 Table 1 sets out the unit root test statistics using the augmented Dickey-Fuller procedure for Australian quarterly data over the period September 1969 to December 1994.

Insert Table 1 about here.

3.19 Table 2 gives the parameter estimates and t statistics for regressions (3) and (4) for this data.

Insert Table 2 about here

3.20 Applying the procedure to the unit root test statistics in Tables 1 and 2 gives the following results:

3.20.1 *Logarithm of the Consumer Price Index*

3.20.1.1 For the variable logCPI, the logarithm of the Consumer Price Index, ϕ_3 does not reject the null hypothesis $\alpha_0\neq0$, α_2=0, α_1=0 so the series has a unit root. ϕ_1 rejects the null α_0=0, α_1=0 suggesting the drift is significant. This is confirmed by the regression (3) where the estimate of α_0 is 0.0244 with a t statistic of 2.94 which is significant at the 0.4% level.

3.20.1.2 For the variable ΔLogCPI, the first difference of logCPI, ϕ_3 rejects the null hypothesis $\alpha_0\neq0$, α_2=0, α_1=0 for the differences. The τ_2 test statistic rejects the hypothesis that α_1=0 so there is evidence that the differences are stationary.

3.20.1.3 This analysis suggests that the logarithm of the CPI is integrated of order 1 and differences in the logarithm of the CPI are a stationary series.

3.20.2 *Logarithm of the Share Price Index*

3.20.2.1 For the variable LogSPI, the logarithm of the Share Price Index, ϕ_3 does not reject the null hypothesis $\alpha_0\neq0$, α_2=0, α_1=0 so the series has a unit root with no trend. ϕ_2 does not reject the null α_0=0, α_2=0, α_1=0 so that this is evidence of no drift. Using the more powerful test with ϕ_1 does not reject the null α_0=0, α_1=0. From the regression (3) the estimate of α_0 is 0.0547 with a t statistic of 0.52 which is not significant.

3.20.2.2. For ΔLogSPI, ϕ_3 rejects the null hypothesis that $\alpha_0\neq0$, α_2=0, α_1=0 for the differences. The τ_2 test statistic rejects the hypothesis that α_1=0 so there is evidence that the differences are stationary. From regression (4) the estimate of α_0 is 0.0128 and of α_2 is 0.0002 and neither of these is significant. We should not however conclude that the expected return on the SPI is zero. The standard statistical testing procedure used for unit roots is not necessarily powerful enough to differentiate between a small positive expected return and a zero return. The SPI should have a positive expected return for the index to grow over time. In this case the variability in the growth rate is high and the statistical test is not powerful enough to differentiate a small positive return from a zero return.

3.20.2.3 Based on these results we conclude that the logarithm of the SPI is integrated of order 1 and differences in the logarithm of the SPI are a stationary series. Although the statistical results suggest that the drift in the difference of log(SPI) is not significantly different from zero, we would not expect α_0 to be zero from an economic point of view. The variability in equity returns is too great to confirm a statistically small positive return different from zero.

3.20.3 Logarithm of Average Weekly Earnings

3.20.3.1 Now consider the variable LogAWE, the logarithm of Average Weekly Earnings. The test statistic ϕ_3 does not reject the null hypothesis $\alpha_0 \neq 0$, $\alpha_2=0$, $\alpha_1=0$ so the series has a unit root with no trend. ϕ_2 does not reject the null $\alpha_0=0$, $\alpha_2=0$, $\alpha_1=0$ so this does not reject the zero drift hypothesis. Using the more powerful test with ϕ_1 does reject the null $\alpha_0=0$, $\alpha_1=0$ suggesting that the drift is significant. From the regression (3) the estimate of α_0 is 0.1182 with a t statistic of 3.04 which confirms this.

3.20.3.2 For ΔLogAWE, ϕ_3 does not reject the null hypothesis $\alpha_0 \neq 0$, $\alpha_2=0$, $\alpha_1=0$ for the differences which suggests that the differences have a unit root. However the τ_2 test statistic rejects the hypothesis that $\alpha_1=0$ and this is evidence that the differences are stationary. Note that if the differences in the logarithm of AWE are not stationary then this means that a random shock to the continuously compounding growth rate of AWE would be permanent. A model with this feature would not be sensible since it would allow the continuously compounding growth rate to become arbitrarily large or small.

3.20.3.3. Thus, the logarithm of AWE is most likely integrated of order 1 although the statistical tests suggest it could be of higher order. From an economic perspective, if the logarithm of the CPI is assumed to be integrated of order 1, then the logarithm of AWE should also be integrated of order 1. A higher order of integration of logAWE than of logCPI would mean that these series could not be linked, as economic reasoning would suggest.

3.20.4 Logarithm of the Share Index Dividend Series

3.20.4.1 For LogSDiv, the logarithm of the Share Index Dividend Series, the test statistic ϕ_3 does not reject the null hypothesis $\alpha_0 \neq 0$, $\alpha_2=0$, $\alpha_1=0$ so the series has a unit root with no trend. ϕ_2 does not reject the null $\alpha_0=0$, $\alpha_2=0$, $\alpha_1=0$ so this does not reject the zero drift hypothesis. Using the more powerful test with ϕ_1 does not reject the null $\alpha_0=0$, $\alpha_1=0$ suggesting that the drift is not significant. From the regression (3) the estimate of α_0 is 0.0940 with a t statistic of 1.188 which confirms this.

3.20.4.2 For the differences in the logarithm of the Share Index Dividend Series, ΔLogSDiv, the test statistic ϕ_3 does reject the null hypothesis $\alpha_0 \neq 0$, $\alpha_2=0$, $\alpha_1=0$ which is evidence that the differences in the series are stationary without trend. The τ_2 test statistic rejects the hypothesis that $\alpha_1=0$ so this is further evidence that the differences are stationary.

3.20.4.3 We conclude that the logarithm of the dividend series is integrated of order 1 and the differences in the series have no drift.

3.20.5 Dividend yields on the Share Price Index

3.20.5.1 For SDyields, the Dividend yields on the Share Price Index, ϕ_3 does not reject the null hypothesis $\alpha_0\neq0$, $\alpha_2=0$, $\alpha_1=0$ so the series has a unit root with no trend. ϕ_2 does not reject the null $\alpha_0=0$, $\alpha_2=0$, $\alpha_1=0$ so this does not reject the zero drift hypothesis. Using the more powerful test with ϕ_1 does not reject the null $\alpha_0=0$, $\alpha_1=0$ suggesting that the drift is not significant. However regression (3) shows an estimate for α_0 of 0.849 and this is significant.

3.20.5.2 For the differences, ΔSdyields, the test statistics ϕ_3 does reject the null hypothesis $\alpha_0\neq0$, $\alpha_2=0$, $\alpha_1=0$ for the differences of the series which is evidence that the differences are stationary. The τ_2 test statistic rejects the hypothesis that $\alpha_1=0$ so this is further evidence that the differences are stationary.

3.20.5.3 Thus it appears that the dividend yield series is integrated of order 1 and the differences in the series are stationary with zero drift. Clearly this conclusion means that dividend yields can drift to arbitrarily large or small values. For the historical data set used for the statistical tests, we do not conclude that dividend yields are mean-reverting as might generally be expected. The tests used here do not have the power to detect a close to stationary model versus a non-stationary model for this series.

3.20.6 Interest Rates

3.20.6.1 For the interest rate series, BB90 (90 day bank bill yields), TB2 (2 year Treasury bond yields), TB5 (5 year Treasury bond yields), and TB10 (10 year Treasury bond yields), the same test statistics are significant for all of the interest rate series. ϕ_3 does not reject the null hypothesis $\alpha_0\neq0$, $\alpha_2=0$, $\alpha_1=0$ so this is evidence that each of the series has a unit root with no trend. ϕ_2 does not reject the null $\alpha_0=0$, $\alpha_2=0$, $\alpha_1=0$ so this does not reject the zero drift hypothesis. Using the more powerful test with ϕ_1 does not reject the null $\alpha_0=0$, $\alpha_1=0$ suggesting that the drift is not significant.

3.20.6.2 For the differences in the interest rate series, ΔBB90, ΔTB2, ΔTB5 and ΔTB10, ϕ_3 does reject the null hypothesis $\alpha_0\neq0$, $\alpha_2=0$, $\alpha_1=0$ for all interest rate series which is evidence that the differences of each of the series is stationary without trend. The τ_2 test statistic rejects the hypothesis that $\alpha_1=0$ so this is further evidence that the differences of the series are stationary.

3.20.6.3 Thus each of the interest rate series appears to be integrated of order 1 so the changes in yields are stationary with no drift. This means that interest rate levels can drift to arbitrarily high, or low, levels over long time periods. This may not be considered a satisfactory model of interest rates in practice. A regime switching model using the techniques in Harris (1997), with a mean reverting regime for higher interest rates reflecting government policy to control interest rate levels, may be a more realistic model. Even though the interest rates could not be statistically distinguished from a random walk, if such a model was adopted for interest rates then it would be necessary to consider a cointegrated model. The issue of cointegration of interest rates and inflation rates is examined for this data in section 4 of this paper.

3.21 Phillips and Perron (1988) have proposed non-parametric procedures for testing for unit roots with more general assumptions concerning εt than the assumptions for the Dickey-Fuller and augmented Dickey-Fuller tests. Table 3 sets out the equivalent test statistics to those in Table 1 using the Phillips-Perron test procedures. These were calculated using the procedures in Shazam (1993).

Insert Table 3 about here.

3.22 The conclusions already drawn for the SPI and for the interest rate series are supported by these test statistics. There are however some differences apparent for the other series. For the logCPI series the Phillips-Perron $\phi 3$ rejects the null hypothesis $\alpha 0 \neq 0$, $\alpha 2=0$, $\alpha 1=0$ and the hypothesis that $\alpha 1=0$ is not rejected. The conclusion is that the series has a unit root with a trend. The differences in the series are stationary. Similar conclusions are reached for logAWE using the Phillips-Perron statistics. In the case of LogSDiv, the logarithm of the dividends series, the Phillips-Perron statistics suggest that this series is difference stationary with drift. Dividend yields are difference stationary without drift.

3.23 So far the data period used has been common to all the series covering the period September 1969 to December 1994. Some of the series are available for longer time periods. Tables 4 and 5 report unit root test statistics for these longer time periods for the relevant series.

Insert Table 4 about here.

Insert Table 5 about here.

3.24 For the period March 1939 to March 1995 Tables 4 and 5 provide support for the hypothesis that logSPI is difference stationary with drift. This longer period of data provides a more reliable estimate of the drift. The conclusion is that the logSPI is difference stationary with positive drift. There is evidence in Table 4 that logAWE and logCPI are integrated of a higher order than 1 but the results in Table 5 suggest that they are integrated of order 1.

3.25 Because the data used are quarterly it is necessary to test for seasonal integration. In quarterly data there could be a bi-annual or annual frequency seasonal unit root as well as the quarterly unit root tested for already. Hylleberg et al (1990) develop tests and test statistics for seasonal unit roots. Shazam (1993) provides procedures for implementing these tests. These procedures were applied and bi-annual and annual unit roots are convincingly rejected for all of these series.

3.26 It is worth noting that structural breaks in any series can result in a stationary series appearing to have a unit root. This will lead to differencing the data when a model using the levels of the data and explicitly capturing the structural break would be more appropriate. Differencing series results in the loss of information about the long run level of the series so that care has to be taken to ensure that the series is not stationary.

3.27 It could be argued that deregulation of financial markets during the 1980's resulted in a structural break in many of the series. For instance the method used to sell government securities changed during this period and the bond market became more active. The requirements for life insurance companies, superannuation funds and banks to hold government securities were also relaxed. During this period an imputation tax system was introduced for share investments. All of these factors could well have resulted in structural changes in rates of return and the levels of the series used in this study.

3.28 We have presented the results from applying the standard procedures used in the econometrics literature to determine if a time series is stationary or not. The standard null hypothesis used in these unit root tests is that the series are non-stationary. This is the basis of the test procedures reported in this section of the paper. For purposes of actuarial modelling some caution should be used before adopting the results of these unit root tests. It will be important to ensure that economic reasoning can be used to justify the assumptions used in the model. Bearing in mind some of the cautions raised in sections 3.26 and 3.27 as well as the discussion of the unit test results, the development of an actuarial model will require further analysis beyond the standard unit root tests reported here.

4. COINTEGRATION

4.1 The differencing operation used to achieve stationarity, often used in developing stochastic investment models for actuarial applications, involves a loss of information about long-run movements in the series. The theory of cointegration explains how to study the inter-relationships between the long-term trends in the series. These long-term trends are differenced away in the standard Box-Jenkins approach. The inter-relationships between the long-term trends in the series can be interpreted as equilibrium relationships between the series.

4.2 In financial markets informed investors act quickly on new information particularly when transaction costs are low and markets are liquid. Financial markets can be out of apparent equilibrium as evidenced by speculative bubbles that occur when the share market booms and subsequently crashes even though these events are consistent with rational expectations. Economic systems are less likely to be in equilibrium since friction and price stickiness in goods and labour markets can cause the adjustment process to equilibrium to occur over an extended time frame. This suggests that if equilibrium relationships exist between financial and economic variables then these will only be detected by examining data over long time periods.

4.3 Rates of return on different investments would be expected to have long run equilibrium relationships determining their relative values. For example the spread between the return on a short term investment and the return on a longer term investment should fluctuate around some long term relationship that reflects the risk premium investors require for the longer term investment over the shorter term investment. If a long term relationship does hold then the difference between the returns should have a stationary distribution. The rates of return themselves might not be stationary but a linear combination of them will be stationary if such a long run equilibrium holds. Rates of return adjusted for expected rates of inflation, referred to as “real” rates of return as compared with nominal rates of return, might also be expected to have a stationary distribution.

4.4 Most actuaries assume that there is a relationship between equity returns and inflation. This assumption is usually implicit in the use of “real” rates of return for projecting asset values and for valuation purposes. If a constant “real” rate of return is used then this implicitly assumes that asset returns are perfectly correlated with inflation. The Wilkie model uses inflation as the main factor driving asset returns. Investment model studies by Carter

(1991) and Harris (1995) include results derived from fitting Wilkie's model to Australian data and find no statistically significant empirical relationship between equity returns and rates of inflation. This conflict between often used actuarial assumptions and empirical results clearly requires investigation since it will be fundamental to investment modelling and modelling the interaction between liabilities and assets of insurance companies and pension (superannuation) funds.

4.5 It is important to recognise that equity values and inflation can have a long-run equilibrium relationship and for no statistically significant relationship between equity returns and rates of inflation to exist. This could be the case if the series are co-integrated. Rates of inflation and equity (capital) returns are differences in the logarithm of the level of the inflation index and differences in the logarithm of the equity index respectively. These rates of change in the levels of the indices might appear to have no statistical relationship even though the levels of the indices might be co-integrated with a long run equilibrium relationship. Each of the index series would be difference stationary containing a unit root, consistent both with the notion of market efficiency and with studies of Australian data such as Carter (1991), Harris (1994) and the results in this paper.

4.6 If variables are non-stationary but an equilibrium relationship represented by a linear combination of the variables exists such that this linear combination is stationary then the variables are said to be co-integrated. Engle and Granger (1987) suggested the concept of cointegration and developed tests for cointegration. The concept of cointegration captures the notion that two or more series "move together" in some fashion. The series have common stochastic trends.

4.7 Testing for cointegration between any two series, where there is only one co-integrating linear combination determining the equilibrium relationship between the series, requires only the unit root tests used earlier to determine the order of stationarity of the investment data. Consider two series xt and yt that are integrated of order 1 so that they are difference stationary. If a long-term (linear) relationship exists between these then xt-βyt, for some constant β, will be stationary. If xt is regressed on yt and there is a long run equilibrium relationship between them, then the residuals from this regression will not have a unit root. Thus for these residuals the null hypothesis of a unit root should be rejected if the series are co-integrated. Otherwise there is no evidence of cointegration.

4.8 Table 6 reports the results of unit root cointegration tests for bi-variate series from Australian data using augmented Dickey Fuller tests and Table 7 reports the results using Phillips-Perron tests. From the tests carried out earlier in this paper all of the series used were integrated of order 1. The results in Tables 6 and 7 were calculated using procedures in Shazam (1993). They consider each of the bi-variate series over the longest time period available and also for shorter time periods.

Insert Table 6 about here.

Insert Table 7 about here.

4.9 For this data frequency there is no evidence that any of the bi-variate series considered, other than the 90 day bank bill yield and the 10 year Treasury bond yield, are co-integrated. In all cases other than for these two interest rates the test statistics for both ADF and Phillips-Perron tests given in Tables 6 and 7 do not reject the null hypothesis of a unit root. Thus there is no evidence that the SPI and the CPI 'move together' nor that share index dividends and the CPI 'move together'. It is encouraging to find that the long and short interest rate are co-integrated since this is supported by the results of Ang and Moore (1994).

5. ERROR CORRECTION MODELS

5.1 If a number of series are co-integrated then they have common stochastic trends and move together through time following a long-run equilibrium. This long run equilibrium is disturbed by random shocks that are short term or temporary effects. The series eventually adjusts for these. This short term adjustment process is referred to as an error correction mechanism. Engle and Granger (1987) proved that for any co-integrated series an error correction model exists. The error correction model captures both the short term departures from the long run equilibrium and the long run equilibrium in the model structure.

5.2 Investment models should incorporate error-correction mechanisms to ensure an equilibrium exists in the model. In the data used in the analysis in this paper the long and short run interest rates are co-integrated and should be modelled using an error correction mechanism. Otherwise if differences in interest rates are modelled as stationary variables with no error-correction mechanism then the levels of interest rates will have stochastic trends and 'shocks' to interest rates will be permanent. In such a model interest rates could 'wander' off to arbitrarily high and low levels in a manner inconsistent with the historical data.

5.3 If the short interest rate at time t is denoted by $BB90_t$ and the long interest rate at time by $TB10_t$, both of which are non-stationary, then the previous analysis indicates that a regression of $TB10_t$ on $BB90_t$ results in stationary errors (under standard unit root tests). This means that $TB10_t - (\gamma_1 + \gamma_2\ BB90_t)$ is a stationary process representing the long run (linear) relation between the two series. This equilibrium relationship should be incorporated in an interest rate model using an error correction mechanism. Standard stationary time series modelling techniques will not detect the long-run relationship captured in the stationary error correction mechanism. It is worth stressing that ignoring any cointegrating relationships ignores important information about long-run equilibrium relationships between the series.

5.4 For the interest rates the cointegrating vector $(TB10)_t$ - 4.196 - $0.5905(BB90)_t$ was found to be stationary. It represents the estimated long term linear relationship. The error correction model was estimated to be

$$\Delta(TB10)_t = 0.2325\Delta(BB90)_t - 0.1123((TB10)_{t-1} - 4.1960 - 0.5905(BB90)_{t-1})) + u_t$$

where u_t is stationary. Thus the stochastic trend in $(TB10)_t$ is 23.25% of the $(BB90)_t$ stochastic trend and 11.23% of the previous period's disequilibrium. Table 8 provides the regression details.

Insert Table 8 about here.

5.5 The conclusions that can be drawn from this analysis of co-integrating, or 'long-run' equilibrium, relationships in the Australian returns data are that, with the exception of the interest rate series, the analysis finds no strong evidence that such equilibrium relationships exist between the series analysed. This has implications for the structure of stochastic investment models. It will be important to incorporate an equilibrium structure for interest rates in the model but differences in the logarithms of the SPI, CPI and dividends can be used in the model as stationary variables without the need to incorporate any specific equilibrium between these series. This also adds to the empirical evidence for Australian data that there are no strong relationships between inflation and equity returns.

6. STATE SPACE MODELS

6.1 Carter (1991), Wilkie (1986, 1995) and Thomson (1996) use transfer functions to develop their models. This approach allows the estimation of a cascade structure for a stochastic investment model with causality in one direction assumed. The main driving variable in these models is the rate of inflation. It should be noted that the Wilkie model could be written as a VARMA model where feedback is not allowed. In this case the model is a particular case of the more general VARMA models.

6.2 Transfer functions were examined in this research. The results are not reported in any detail here since it was found that after fitting these models there was evidence of feedback between the different variables. This means that transfer functions will not adequately capture the relationship between the different series since they impose a uni-directional causality that is not supported by the empirical data.

6.3 An alternative model is the vector autoregressive or VAR model. These models are used in practice for asset models and have the advantage that they allow for feedback. VAR models were fitted and it was found that too many lags were required and the models were difficult to interpret. Introducing a moving average term into these models is equivalent to an infinite number of auto-regressive terms so that a vector autoregressive moving average (VARMA) model should provide a more parsimonious model than a VAR model.

6.4 State space models provide an alternative method of representing a stochastic investment model. They have an equivalent (VARMA) representation which has fewer lagged variables than the VAR models. Transfer function models are nested in the VARMA models. A state space model can be written as a state equation:

$$\mathbf{z}_{t+1} = \mathbf{F}\,\mathbf{z}_t + \mathbf{G}\,\mathbf{e}_{t+1}$$

and an observation equation:

$$\mathbf{y_t} = \mathbf{H}\,\mathbf{z_t}$$

where $\mathbf{y_t}$ are actual observations at time t, $\mathbf{z_t}$ is the state of the model at time t, **F**, **G** and **H** are matrices of parameters and $\mathbf{e_t}$ is a vector of mean zero, serially uncorrelated disturbances with covariance matrix Σ. The statistical package SAS was used to fit state space models using its state space procedure which selects the best model using the Akaike information model selection criteria. SAS selects the best number of lags of the variables to use in the state vector of the statespace model using the Akaike information model selection criteria by considering a range of lags. Full details of the how SAS selects the best model is found in the SAS/ETS Users Guide (SAS Institute, 1993). The series used in the model are the relevant state variables and these are assumed to be observed without error.

6.5 Returns and inflation

To examine the relationships between asset returns and inflation, state space models were fitted using each of the individual asset returns series and inflation. Models were fitted to the growth rate of the equity index, the growth rate of dividends, the 10 year bond rate and the rate of inflation. These models will also allow a comparison with transfer function models.

6.5.1 Equity index (SPI) and inflation (CPI)

The following state space model for equity index and inflation rates was fitted as the best model using the Akaike information criteria for the quarterly data from September 1948 to March 1995:

$$\mathbf{z}_t = \begin{bmatrix} y_t \\ x_t \\ y_{t+1|t} \end{bmatrix}, \ \mathbf{F} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0.905 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.432 & 0 \end{bmatrix}, \ \mathbf{e}_{t+1} = \begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix},$$

with

$$\Sigma = \operatorname{var}\begin{bmatrix} n_{t+1} \\ e_{t+1} \end{bmatrix} = \begin{bmatrix} 8.39\text{x}10^{-5} & -7.18\text{x}10^{-5} \\ -7.18\text{x}10^{-5} & 8.84\text{x}10^{-3} \end{bmatrix}.$$

where the variables are the differences in the logarithms of the series, or the continuously compounding returns, adjusted for the mean of the series as follows:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} (1-B)\mathbf{Y}_t - 0.0153 \\ (1-B)\mathbf{X}_t - 0.0161 \end{bmatrix}$$

with $\mathbf{Y}_t = \log(\text{CPI})_t$, $\mathbf{X}_t = \log(\text{SPI})_t$, and $y_{t+1|t} = y_{t+1} - e_{t+1}$ is the predicted value for time t+1 conditional on information at time t. Note that the vectors $\mathbf{e}_{t+1}$ are assumed to be a sequence of independent normally distributed random vectors with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$.

From the covariance matrix the estimated standard deviations of the residuals after fitting the model are 0.0092 for the quarterly continuously compounding rate of inflation and 0.094 for the quarterly continuously compounding rate of growth of the SPI with a correlation between the residuals of -0.0834.

The parameter estimates were:

Parameter	Estimate	Std. Error	t value
F(3,3)	0.905	0.041	22.326
G(3,1)	0.432	0.066	6.563

For simulation studies such as in asset and liability modelling it is important to recognise that this model does not capture parameter or model uncertainty and has been calibrated to historical data over the time period September 1948 to March 1995. The variances in asset returns and rates of inflation are assumed to be homoscedastic in this model. The model can be written as an equivalent VARMA model as follows:

$$\begin{bmatrix} y_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} 0.905 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix} + \begin{bmatrix} -0.473 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e_t \\ n_t \end{bmatrix}$$

Note that using this modelling approach the best model for $\log(SPI)_t$ is a random walk with drift and an ARMA model is required for $\log(CPI)_t$. This model is very different to that suggested by Wilkie.

6.5.2 Equity dividends and inflation (CPI)

The following state space model for equity dividends and inflation was fitted as the best model using the Akaike information criteria for the quarterly data from September 1967 to December 1994:

$$\mathbf{z}_t = \begin{bmatrix} x_t \\ y_t \\ y_{t+1|t} \\ y_{t+2|t} \\ y_{t+3|t} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.030 & 0.340 & -0.236 & 0.481 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0.258 \\ -0.021 & 0.382 \\ 0 & 0.440 \end{bmatrix},$$

$$\mathbf{e}_{t+1} = \begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix}, \Sigma = \mathrm{Var}\begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix} = \begin{bmatrix} 4.26\text{x}10^{-3} & 4.93\text{x}10^{-5} \\ 4.93\text{x}10^{-5} & 6.22\text{x}10^{-5} \end{bmatrix}.$$

with

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} (1-B)\mathbf{X}_t - 0.0208 \\ (1-B)\mathbf{Y}_t - 0.0178 \end{bmatrix}$$

and

$$\mathbf{X}_t = \log(\mathrm{DIVS})_t,\ \mathbf{Y}_t = \log(\mathrm{CPI})_t.$$

The parameter estimates were:

Parameter	Estimate	Std. Error	t-value
F(5,1)	0.030	0.010	3.038
F(5,2)	0.340	0.121	2.800
F(5,3)	-0.236	0.082	-2.864
F(5,4)	0.481	0.150	3.208
G(3,2)	0.258	0.091	2.837
G(4,1)	-0.021	0.010	-2.057
G(4,2)	0.382	0.089	4.290
G(5,2)	0.440	0.089	4.971

This model shows how log(DIVS) and log(CPI) are interrelated. An equivalent VARMA model can be readily developed from the above state space model.

6.5.3. 10-year treasury bond rates (TB10) and CPI

Using the quarterly series from March 1958 to December 1994 the state space model was:

$$\mathbf{z}_t = \begin{bmatrix} x_t \\ y_t \\ y_{t+1|t} \\ y_{t+2|t} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.675 & -0.308 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0.126 \\ 0.382 & 0.182 \end{bmatrix},$$

$$\mathbf{e}_{t+1} = \begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix}, \Sigma = \mathrm{Var}\begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix} = \begin{bmatrix} 7.15\mathrm{x}10^{-5} & 5.09\mathrm{x}10^{-6} \\ 5.09\mathrm{x}10^{-6} & 7.79\mathrm{x}10^{-5} \end{bmatrix}.$$

with

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} (1-B)\mathbf{X}_t - 0.0217 \\ (1-B)\mathbf{Y}_t - 0.0147 \end{bmatrix}$$

and

$$\mathbf{X}_t = (\mathrm{TB10})_t,\ \mathbf{Y}_t = \log(\mathrm{CPI})_t.$$

The parameter estimates were:

Parameter	Estimate	Std. Error	t-value
F(4,3)	0.675	0.042	16.146
F(4,4)	-0.308	0.057	-5.448
G(3,2)	0.126	0.056	2.245
G(4,1)	0.382	0.073	5.232
G(4,2)	0.182	0.052	3.516

The equivalent VARMA model fit is given by the following equation.

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -0.308 \end{bmatrix}\begin{bmatrix} x_t \\ y_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.675 \end{bmatrix}\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.434 \end{bmatrix}\begin{bmatrix} e_t \\ n_t \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 \\ 0.382 & -0.454 \end{bmatrix}\begin{bmatrix} e_{t-1} \\ n_{t-1} \end{bmatrix}$$

These bi-variate models capture the relationship between these variables and assist in understanding the nature of these series and their interrelationships. However it can be seen that the models for the inflation series differ in each of the above models. This suggests that a model incorporating all of the series could provide more information about the best model for inflation since it will incorporate the interrelationships between the series. Such a model was fitted and the resulting model was complex and difficult to interpret so it has not been set out in this paper.

6.6 Other models

Models were also fitted to the SPI and the dividend series as well as the SPI and the 10 year bond yield to examine the relationships between these series.

6.6.1 Equity index (SPI) and equity dividends (DIVS)

The best state-space model using the quarterly series from September 1967 to December 1994 was found to be:

$$\mathbf{z}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{e}_{t+1} = \begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix},$$

$$\Sigma = \mathrm{Var}\begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix} = \begin{bmatrix} 4.26\mathrm{x}10^{-3} & 5.67\mathrm{x}10^{-4} \\ 5.67\mathrm{x}10^{-4} & 1.28\mathrm{x}10^{-2} \end{bmatrix}.$$

where

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} (1-B)\mathbf{X}_t - 0.0208 \\ (1-B)\mathbf{Y}_t - 0.0177 \end{bmatrix}$$

and

$$\mathbf{X}_t = \log(\mathrm{DIVS})_t,\ \mathbf{Y}_t = \log(\mathrm{SPI})_t.$$

The model indicates that these series are random walks with drifts and correlated errors. In this case parameter estimates will be more efficient in a model that includes both series. Even though the statistical evidence supports random walk models for both series the correlation of the errors means that considering the two series simultaneously pools information. Note that the model is for an equity dividend index and not for a dividend yield. The dividend yield is given by the difference in the dividend index divided by the value of the share price index. Models that assume that the dividend yield is stationary and mean-reverting will not necessarily be consistent with this fitted model. Which approach is better is a matter for future research.

6.6.2 Equity index (SPI) and 10-year treasury bond rates (TB10)

The best state-space model for the quarterly series from March 1958 to December 1994 was:

$$\mathbf{z}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 0.982 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{e}_{t+1} = \begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix},$$

$$\Sigma = \mathrm{var}\begin{bmatrix} e_{t+1} \\ n_{t+1} \end{bmatrix} = \begin{bmatrix} 2.71\mathrm{x}10^{-6} & -3.67\mathrm{x}10^{-5} \\ -3.67\mathrm{x}10^{-5} & 1.05\mathrm{x}10^{-2} \end{bmatrix}.$$

where

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} (1-B)\mathbf{X}_t - 0.0217 \\ (1-B)\mathbf{Y}_t - 0.0177 \end{bmatrix}$$

and

$$\mathbf{X}_t = (\mathrm{TB10})_t,\ \mathbf{Y}_t = \log(\mathrm{SPI})_t.$$

The parameters were:

Parameter	Estimate	Std. Error	T-Value
F(1,1)	0.982	0.016	63.029

Note that the fitted 10 year bond yield model is close to a random walk with drift and the equity index is a random walk with drift.

6.7 Summary

6.7.1 These models have all been bi-variate models based on quarterly Australian data. Ignoring heteroscedasticity, they provide support for the random walk model for the equity index. They also provide support for modelling the difference in interest rates, and not the level of the series, as a stationary series. These features are not found in many of the stochastic models that have details available in the public domain.

6.7.2 For asset liability studies it will be important to have a model to project the equity returns, dividends, inflation and interest rates as a multi-variate system. As noted earlier such a model appears to be rather complex and difficult to interpret. In the state space approach the parameters in F and G can be allowed to be time varying. The models can incorporate parameter uncertainty for forecasting purposes. Using parameter estimates from historical data for forecasting or model projection ignores any parameter uncertainty and is likely to understate the future uncertainty.

6.7.3 The Kalman filter maximum likelihood approach can be used with state space models to estimate model parameters. This leads to an estimation procedure that allows recursive model estimation and updating.

6.7.4. It should also be noted that tests of the assumptions of the model concerning the residuals have not been performed for these models. The results are based on the assumptions of i.i.d. and normally distributed errors.

CONCLUSIONS

7.1 This paper has set out the results of research into the structural features of a stochastic investment model for actuarial applications using Australian data. This analysis is fundamental to the construction of a soundly based model. It has analysed Australian investment data using a quarterly time period. It has formally tested stationarity of all the series and examined which series are cointegrated and therefore maintain a long run equilibrium relationship. It has also examined the appropriateness of transfer function models that assume one way causality between series using Australian investment data.

7.2 The results of the research suggest that the stationary variables in the Australian investment quarterly data are the rate of (continuously compounding) growth in the Share Price Index (SPI), the rate of (continuously compounding) growth in the Consumer Price Index (SPI), the rate of (continuously compounding) growth in a Dividend index representing the dividends on the SPI, and differences in the interest rate series. The statistical analysis did not provide evidence that the interest rate levels were stationary.

7.3 The cointegration tests indicated a long-run equilibrium relationship exists between the interest rates whereas there was no evidence to support such a relationship between equity values, as measured by the SPI and a Dividend index, and the level of the inflation index (CPI).

7.4 Transfer functions models were fitted to the various series and inflation but they were not found to capture the relationships between these series. State space models for the different series and inflation were fitted to allow a comparison with transfer function models fitted by other researchers.

7.5 This research highlights some important lessons for those wishing to construct and use stochastic investment models. It indicates the type of analysis that should be the foundation of an analysis of the series to be used in these models and the nature of the relationships that should be included. These matters are fundamental to the construction of stochastic investment models.

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APPENDIX A

AUSTRALIAN INVESTMENT DATA - SUMMARY STATISTICS

Different series were available for different time periods. In the analysis in the paper the longest time periods available for the series have been used where possible. Statistics for these series over different time periods are summarised in this Appendix.

(1) Consumer Price Index - all groups (CPI) and All Ordinaries Share Price Index (SPI)

Quarterly data were available for these series for the period September 1948 to March 1995. Table A1 sets out summary statistics for these indices, the logarithm of the index and the change in the logarithm.

Table A1. Summary statistics of CPI and SPI, logarithm of CPI and SPI and first differences of logarithms of CPI and SPI.

Statistics	N	Min	Max	Mean	St Dev	Skewness	Excess Kurtosis
CPI	187	6.70	114.70	39.1064	34.0734	1.0008	-0.4688
log(CPI)	187	1.9021	4.7423	3.2964	0.8573	0.3615	-1.2894
Δlog(CPI)	186	-0.0087	0.0704	0.0153	0.0132	1.1935	2.0125
SPI	187	84.60	2238.70	561.5283	567.3915	1.3879	0.6055
log(SPI)	187	4.4379	7.7137	5.8806	0.9354	0.3983	-0.9342
Δlog(SPI)	186	-0.5728	0.2613	0.0161	0.0940	- 1.6993	8.7947

Note the negative skewness and high kurtosis for the continuously compounding return on the SPI - given by the variable **Δlog(SPI).**

(2) Consumer Price Index - all groups (CPI) and Share Dividends (DIVS)

Quarterly data for the period September 1967 to December 1994 was available for the CPI. The Dividend yield series is the Melbourne weighted (M.W.) series from September 1967 to December 1982. This was merged with the Australian dividend yield (A.Y.) series, which is available from September 1983 to March 1995, by taking 2/3M.W.+1/3A.Y. for March 1983 and 1/3M.W.+2/3A.Y. for June 1983. The share dividend series (DIVS) is derived as the product of the SPI and the dividend yield for each quarter. It represents an annualised amount of dividends paid over the prior 12 months. Table A2 sets out summary statistics for these series, the logarithms of the series and the differences in the logarithms of the series.

Table A2. Summary statistics of CPI and DIVS, logarithm of CPI and DIVS and first differences of logarithms of CPI and DIVS.

Statistics	N	Min	Max	Mean	St Dev	Skewness	Excess Kurtosis
CPI	110	16.20	112.80	56.9036	33.2526	0.3283	-1.3353
log(CPI)	110	2.7850	4.7256	3.8406	0.6678	-0.2351	-1.3759
Δlog(CPI)	109	-0.0046	0.0566	0.0178	0.0116	0.7168	1.0666
DIVS	110	747.40	9398.25	3526.76	2603.95	0.8094	-0.6369
log(DIVS)	110	6.6166	9.1483	7.8808	0.7802	0.0526	-1.3005
Δlog(DIVS)	109	-0.1987	0.2132	0.0208	0.0653	-0.3005	1.2734

(3) Interest Rates

Quarterly interest rate data (% p.a.) are available for 90-day Bank Bills (BB90) from September 1969 to December 1994, for 5-year Treasury Bonds (TB5) from June 1969 to December 1994 and for 10-year Treasury Bonds (TB10) from March 1958 to December 1994. The summary statistics of BB90, log(BB90), $\Delta\log(BB90)_t$, TB5, $\Delta(TB5)_t$, TB10, and $\Delta(TB10)_t$ are given in Table A3.

Table A3 Summary statistics of Interest rates (% p.a.) and first differences of Interest rates.

Statistics	N	Min	Max	Mean	St Dev	Skewness	Excess Kurtosis
BB90	102	4.45	19.95	10.9093	4.1029	0.3310	-0.8313
log(BB90)	102	1.4929	2.9932	2.3148	0.3981	-0.2784	-0.8812
Δlog(BB90)	101	-0.4002	0.6213	0.0034	0.1712	0.6058	1.4652
TB5	103	0.0128	0.0394	0.0253	0.0072	-0.0909	-1.0977
Δ(TB5)	102	-0.0060	0.0050	0.0001	0.0019	-0.1966	1.2405
TB10	148	0.0106	0.0394	0.0216	0.0085	0.2654	-1.3368
Δ(TB10)	147	-0.0056	0.0048	0.00008	0.0014	-0.1179	3.7768

(4) All series

Quarterly data for all series was available from September 1969 to December 1994. Table A4 provides summary statistics for this time period.

Table A4 Summary statistics of all series
Quarterly Data from September 1969 to December 1994

Variable	Mean	St.Dev.	Max	Min	Median	Mode	Skewness	Excess Kurtosis
CPI	60.074	32.462	112.80	17.000	55.300	107.60	0.2375	-1.3631
Log(CPI)	3.9220	0.62386	4.7256	2.8332	4.0128	4.6784	-0.3408	-1.2288
AWE	776.57	398.30	1364.3	176.90	796.33	1000.8	-0.0145	-1.3769
Log(AWE)	6.4806	0.64460	7.2184	5.1756	6.6800	6.9086	-0.6544	-0.8507
SPI	865.01	595.05	2238.7	194.30	603.40	2238.7	0.6797	-1.0008
Log(SPI)	6.5177	0.71137	7.7137	5.2694	6.4026	7.7137	0.1667	-1.4523
SD yields	4.4506	1.1496	7.7300	2.0700	4.5000	5.8500	0.2237	-0.1128
SDiv	3741.5	2584.0	9398.3	861.74	2877.4	9398.3	0.7365	-0.7603
BB90	10.909	4.1029	19.950	4.4500	10.350	15.450	0.3310	-0.8313
TB2	10.185	3.2623	16.400	4.6000	9.9400	15.150	0.0137	-1.1443
TB5	10.465	2.9845	16.400	5.2000	10.030	13.850	-0.0775	-1.0843
TB10	10.648	2.8299	16.400	5.7500	10.180	9.5000	-0.0997	-1.0091

Table A5 Jarque-Bera Asymptotic LM Normality Test
September 1969 - December 1994
Chi-squared 2DF 5% Critical Value 5.99

Variable	Chi-Square Statistic
CPI	8.74*
LogCPI	8.32*
AWE	7.96*
LogAWE	10.27*
SPI	11.97*
LogSPI	9.28*
SD yields	9.47*
SDiv	11.55*
BB90	4.87
TB2	5.60
TB5	5.15
TB10	4.57

*significant at 5% level

Table 1. Test Statistics for Unit Roots – Augmented Dickey-Fuller Regressions

Variable	n	τ_1	ϕ_1	τ_2	ϕ_2	ϕ_3
10% Critical Value		*(-2.57)*	*(3.78)*	*(-3.13)*	*(4.03)*	*(5.34)*
LogCPI	96	-2.5394	5.0773*	-0.14401	3.3658	3.2153
ΔLogCPI	95	-1.9654	1.9387	-3.2620*	3.8593	5.7811*
LogSPI	101	-0.36670	1.0593	-2.5374	2.9977	3.4468
ΔLogSPI	93	-4.0574*	8.2403*	-4.0353*	5.5106*	8.2573*
LogAWE	94	-2.9235*	5.0615*	-1.1134	3.3407	4.2320
ΔLogAWE	94	-1.6971	1.5320	-3.1763*	3.4963*	5.1460
LogSDiv	91	-0.94375	3.2947	-2.2644	3.8328	2.7697
ΔLogSDiv	97	-3.6085*	6.5145*	-3.5827*	4.3077*	6.4580*
SDyields	94	-2.6065*	3.4282	-2.4752	2.4863	3.6984
ΔSDyields	93	-4.2322*	8.9565*	-4.3210*	6.2671*	9.3998*
BB90	95	-2.0651	2.1326	-1.8133	1.4128	2.1190
ΔBB90	93	-4.3252*	9.3662*	-4.5146*	6.8251*	10.225*
TB2	98	-2.1987	2.4757	-2.2523	1.7769	2.6071
ΔTB2	97	-3.5883*	6.4738*	-3.5303*	4.3129*	6.4337*
TB5	98	-1.9812	2.0334	-1.8892	1.3792	1.9985
ΔTB5	96	-3.5858*	6.4526*	-3.6100*	4.4730*	6.6862*
TB10	101	-1.8629	1.9083	-1.3939	1.3380	1.8353
ΔTB10	98	-4.8847*	11.930*	-4.9430*	8.2041*	12.306*

* indicates significant at 10% level, based on the limiting distributions

n is the number of observations, $\boldsymbol{\tau_1}$ is the least squares t statistic for regression (3), $\boldsymbol{\tau_2}$ is the least squares t statistic for regression (4)

Table 2. Tests for Unit Roots - Parameters of Augmented Dickey-Fuller Regressions
(t statistics in brackets beneath the estimate)

		Regression 3			Regression 4		
Variable	p	α_0	α_1	p	α_0	α_1	α_2
LogCPI	5	0.02438	-0.00446	5	0.017387	-0.00175	-0.00006
		(2.94)	(-2.539)		(0.5417)	(-0.1440)	(-0.2255)
ΔLogCPI	5	0.00390	-0.20633	5	0.01406	-0.41302	-0.00011
		(1.744)	(-1.965)		(3.262)	(-3.262)	(-2.724)
LogSPI	0	0.05472	-0.00594	0	0.54693	-0.10166	0.00251
		(0.5161)	(-0.3667)		(2.536)	(-2.537)	(2.598)
ΔLogSPI	7	0.02421	-1.2959	7	0.01283	-1.3302	0.00021
		(1.803)	(-4.057)		(0.4519)	(-4.035)	(0.4551)
LogAWE	7	0.11815	-0.01577	7	0.12763	-0.01766	0.00005
		(3.041)	(-2.923)		(1.513)	(-1.113)	(0.1267)
ΔLogAWE	6	0.00520	-0.29492	6	0.03463	-0.80799	-0.00034
		(1.204)	(-1.697)		(2.952)	(-3.176)	(-2.684)
LogSDiv	10	0.09404	-0.00092	10	0.99212	-0.14792	0.00361
		(1.188)	(-0.9437)		(2.332)	(-2.264)	(2.147)
ΔLogSDiv	3	0.01354	-0.67085	3	0.01587	-0.67512	-0.00004
		(1.789)	(-3.609)		(1.043)	(-3.583)	(-0.1772)
SDyields	7	0.84903	-0.18277	7	0.91397	-0.17543	-0.00176
		(2.610)	(-2.607)		(2.719)	(-2.475)	(-0.7951)
ΔSDyields	7	0.01025	-1.3678	7	0.13634	-1.4646	-0.00224
		(0.1704)	(-4.232)		(0.9379)	(-4.321)	(-0.9529)
BB90	6	1.3958	-0.12360	6	1.4177	-0.11965	-0.00121
		(1.987)	(-2.065)		(1.963)	(-1.813)	(-0.1463)
ΔBB90	7	0.00596	-1.5594	7	0.58113	-1.7217	-0.01032
		(0.02885)	(-4.325)		(1.167)	(-4.515)	(-1.268)
TB2	3	0.74802	-0.06935	3	0.71168	-0.07914	0.00255
		(2.203)	(-2.199)		(2.061)	(-2.252)	(0.6407)
ΔTB2	3	0.03630	-0.71139	3	0.10645	-0.73037	-0.00130
		(0.3556)	(-3.588)		(0.4620)	(-3.530)	(-0.3400)
TB5	3	0.61876	-0.05561	3	0.61260	-0.06071	0.00111
		(2.011)	(-1.981)		(1.977)	(-1.889)	(0.3311)
ΔTB5	4	0.03165	-0.79979	4	0.16815	-0.86536	-0.00249
		(0.3752)	(-3.586)		(0.8454)	(-3.610)	(-0.7582)
TB10	0	0.54248	-0.04698	0	0.54105	-0.04031	-0.00134
		(1.951)	(-1.863)		(1.939)	(-1.394)	(-0.4767)
ΔTB10	2	0.02894	-0.84539	2	0.15870	-0.88785	-0.00240
		(0.3953)	(-4.885)		(0.9776)	(-4.943)	(0.8958)

Values for p were determined using the procedures in the econometric package Shazam (1993)

Table 3. Test Statistics for Unit Roots -Phillips-Perron Tests (n=101)

Variable	τ_1	ϕ_1	τ_2	ϕ_2	ϕ_3
10% Critical Value	*(-2.57)*	*(3.78)*	*(-3.13)*	*(4.03)*	*(5.34)*
LogCPI	-3.8378*	119.83*	2.2660	98.752*	13.073*
ΔLogCPI	-5.2039*	13.482*	-6.3676*	13.538*	20.307*
LogSPI	-0.3098	0.9029	-2.8153	3.4695	4.3031
ΔLogSPI	-9.8017*	48.052*	-9.8160*	32.128*	48.178*
LogAWE	-4.1848*	48.665*	0.0434	33.830*	9.6581*
ΔLogAWE	-8.4424*	35.636*	-9.9542*	33.037*	49.544*
LogSDiv	-1.1221	6.5966*	-1.5004	4.8385*	1.4252
ΔLogSDiv	-10.607*	56.279*	-10.613*	37.587*	56.374*
SDyields	-3.1195*	4.9308*	-3.0255	3.3472	4.9644
ΔSDyields	-9.1122*	41.529*	-9.4328*	27.813*	41.712*
BB90	-2.7201	3.7059	-2.6124	2.4444	3.6589
ΔBB90	-10.773*	58.023*	-10.775*	38.702*	58.047*
TB2	-1.9289	1.9504	-1.7487	1.2882	1.8453
ΔTB2	-9.1718*	42.071*	-9.1772*	28.106*	42.152*
TB5	-1.8815	1.8962	-1.5613	1.2723	1.7848
ΔTB5	-9.2738*	43.005*	-9.3308*	29.027*	43.541*
TB10	-4.8920*	1.9395	-1.4685	1.3388	1.8608
ΔTB10	-9.0689*	41.130*	-9.1662*	28.013*	42.018*

* indicates significant at 10% level, based on the limiting distributions

Table 4. Test Statistics for Unit Roots - ADF Regressions
Various Periods

Variable	N	τ_1	ϕ_1	τ_2	ϕ_2	ϕ_3
10% Critical Value		*(-2.57)*	*(3.78)*	*(-3.13)*	*(4.03)*	*(5.34)*
Data from March 1939 to March 1995						
LogSPI	223	-0.0369	3.2802	-2.7539	4.9204*	3.9974
ΔLogSPI	210	-4.4210*	9.7739*	-4.4454*	6.5980*	9.8960*
LogAWE	210	-0.6899	2.1214	-2.4390	3.3084	3.0349
ΔLogAWE	210	-2.3994	2.9010	-2.3545	1.9627	2.9216
Data from March 1958 to December 1994						
LogCPI	179	0.18249	3.3767	-1.6925	3.3323	1.5961
ΔLogCPI	180	-3.2500*	5.2817*	-3.2444*	3.5113	5.2666
LogSPI	185	-0.29941	2.8000	-2.4794	3.9967	3.1611
ΔLogSPI	172	-4.0135*	8.0617*	-4.0168*	5.4166*	8.1171*
LogAWE	172	-0.07160	1.4983	-2.6340	3.4014	3.5482
ΔLogAWE	173	-2.6096*	3.7562	-2.6057	2.5520	3.4787
Data from March 1958 to December 1994						
LogCPI	147	-0.51786	1.3032	-2.6342	3.1287	3.4721
ΔLogCPI	141	-2.1290	2.2675	-1.9831	1.5080	2.2608
LogSPI	146	-0.60057	2.3429	-2.1793	3.0718	2.3939
ΔLogSPI	138	-4.3209*	9.3540*	-4.3673*	6.3799*	9.5508*
LogAWE	134	-1.0694	2.1296	-1.4421	1.9822	1.4069
ΔLogAWE	134	-1.6881	1.4316	-1.6724	1.2924	1.9319
TB10	143	-1.4655	1.2807	-1.6998	1.2059	1.6020
ΔTB10	142	-4.4264*	9.8343*	-4.4309*	6.6185*	9.8903*

* indicates significant at 10% level, based on the limiting distributions

Table 5. Test Statistics for Unit Roots - Phillips-Perron Tests
Various Periods

Variable	τ_1	ϕ_1	τ_2	ϕ_2	ϕ_3
10% Critical Value	*(-2.57)*	*(3.78)*	*(-3.13)*	*(4.03)*	*(5.34)*
Data from March 1939 to March 1995 (n=224)					
LogSPI	-0.066319	3.1281	-2.8455	4.9236*	4.2426
ΔLogSPI	-14.151*	100.13*	-14.143*	66.678*	100.02*
LogAWE	0.020279	57.654*	-1.0626	38.751*	0.58240
ΔLogAWE	-11.358*	64.474*	-11.332*	42.781*	64.171*
Data from March 1948 to December 1994 (n=186)					
LogCPI	0.23384	73.724*	-0.64481	49.144*	0.28228
ΔLogCPI	-5.5568*	15.382*	-5.5431*	10.185*	15.278*
LogSPI	-0.32714	2.6717	-2.5693	4.0070*	3.3852
ΔLogSPI	-12.851*	82.575*	-12.827*	54.846*	82.268*
LogAWE	-0.96316	48.872*	-0.52415	32.424*	0.52388
ΔLogAWE	-10.584*	55.994*	-10.604*	37.463*	56.192*
Data from March 1958 to December 1994 (n=147)					
LogCPI	1.5225	75.700*	-2.5431	57.601*	5.4467*
ΔLogCPI	-5.1516*	13.206*	-5.2291*	9.1058*	13.655*
LogSPI	-0.61547	2.2874	-2.2381	3.0804	2.5254
ΔLogSPI	-11.650*	67.861*	-11.611*	44.949*	67.421*
LogAWE	-0.42646	39.283*	-0.52395	26.044*	0.19468
ΔLogAWE	-10.069*	50.672*	-10.041*	33.597*	50.395*
TB10	-1.3495	1.1305	-1.3746	0.90826	1.1456
ΔTB10	-10.920*	-59.631	-10.913*	39.706*	59.559*

* indicates significant at 10% level, based on the limiting distributions

Table 6. Test Statistics for Cointegration - ADF Regression Tests
Various Periods

Variable	n	τ_1	ϕ_1	τ_2	ϕ_2	ϕ_3
10% Critical Value		*(-2.57)*	*(3.78)*	*(-3.13)*	*(4.03)*	*(5.34)*
Data from September 1948 to March 1995						
RSC (SPI-CPI)	186	-2.2308	2.4892	-2.2755	1.7733	2.6588
ΔRSC	186	-3.9832*	7.9760*	-3.9460*	5.2886*	7.8902*
Data from March 1958 to December 1994						
RSC (SPI-CPI)	147	-2.2356	2.5917	-2.2261	1.7159	2.4817
ΔRSC	138	-4.1852*	8.7753*	-4.1839*	5.8690*	8.7864*
R10C(TB10-CPI)	144	-1.8067	1.6823	-1.7801	1.2556	1.8333
ΔR10C	147	-4.5473*	10.377*	-4.5761*	7.0730*	10.571*
Data from September 1967 to December 1994						
RSC (SPI-CPI)	109	-1.7755	1.5808	-1.8020	1.1877	1.7770
ΔRSC	100	-3.5151*	6.1879*	-3.7956*	4.8270*	7.2305*
RSD (SPI-DIVS)	102	-1.9911	1.9934	-2.0944	2.0728	3.0978
ΔRSD	100	-3.8000*	7.2352*	-4.1384*	5.7208*	8.5658*
RDC (DIVS-CPI)	99	-2.0153	2.0389	-2.0644	1.4597	2.1813
ΔRDC	99	-4.5093*	10.167*	-4.4580*	6.7409*	10.111*
Data from September 1969 to December 1994						
RSC (SPI-CPI)	101	-2.1646	2.3788	-2.2573	2.1308	3.1598
ΔRSC	93	-3.9151*	7.6758*	-4.0427*	5.4993*	8.2371*
RSD (SPI-DIVS)	94	-2.2297	2.4956	-2.4545	2.5298	3.7847
ΔRSD	93	-4.2881*	9.1979*	-4.3814*	6.4889*	9.7294*
RDC (DIVS-CPI)	91	-1.9483	1.9080	-2.0843	1.5118	2.2576
ΔRDC	97	-3.7729*	7.1208*	-3.7797*	4.8360*	7.2507*
RB90T10 (BB90-TB10)	95	-3.0086*	4.6227*	-3.3381*	3.7971	5.5976*
ΔRB90T10	93	-4.5139*	10.189*	-4.4986*	6.7474*	10.120*
RB90C (BB90-CPI)	95	-1.9732	1.9849	-1.9528	1.5621	2.3052
ΔRB90C	93	-4.3734*	9.5769*	-4.5307*	6.8770*	10.302*
RT10C (TB10-CPI)	101	-1.4796	1.0999	-1.6161	1.3038	1.9502
ΔRT10C	98	-4.9972*	12.486*	-5.0212*	8.4723*	12.708*

* indicates significant at 10% level

RSC are the residuals from the regression logSPI = $\alpha_0 + \alpha_1$ logCPI

RSD are the residuals from the regression logSPI = $\alpha_0 + \alpha_1$ logSDiv

RDC are the residuals from the regression logSDiv = $\alpha_0 + \alpha_1$ logCPI

RB90T10 are the residuals from the regression BB90 = $\alpha_0 + \alpha_1$ TB10

RB90C are the residuals from the regression BB90 = $\alpha_0 + \alpha_1$ logCPI

RT10C are the residuals from the regression TB10 = $\alpha_0 + \alpha_1$ logCPI

Table 7. Test Statistics for Cointegration - Phillips-Perron Tests
Various Periods

Variable	N	τ_1	ϕ_1	τ_2	ϕ_2	ϕ_3
10% Critical Value		*(-2.57)*	*(3.78)*	*(-3.13)*	*(4.03)*	*(5.34)*
Data from September 1948 to March 1995						
RSC	187	-2.3507	2.7666	-2.3937	1.9552	2.9318
ΔRSC	186	-12.372*	76.536*	-12.345*	51.800*	76.200*
Data from March 1958 to December 1994						
RSC	148	-2.3100	2.7567	-2.3011	1.8269	2.6546
ΔRSC	147	-11.440*	65.442*	-11.400*	43.330*	64.991*
R10C	148	-1.5317	1.2507	-1.5376	1.0544	1.5053
ΔR10C	147	-11.194*	62.657*	-11.207*	41.872*	62.807*
Data from September 1967 to December 1994						
RSC	110	-1.8159	1.6544	-1.8381	1.2285	1.8384
ΔRSC	109	-10.128*	51.297*	-10.122*	34.158*	51.277*
RSD	110	-2.2646	2.5663	-2.2649	1.7906	2.6852
ΔRSD	109	-10.423*	54.332*	-10.413*	36.145*	54.207*
RDC	110	-1.9553	1.9130	-1.9500	1.2701	1.9015
ΔRDC	109	-11.337*	64.269*	-11.283*	42.449*	63.674*
Data from September 1969 to December 1994						
RSC	102	-2.1995	2.4551	-2.2783	2.1503	3.1902
ΔRSC	101	-9.6874*	46.938*	-9.7916*	31.968*	47.938*
RSD	102	-2.7287	3.7547	-2.7584	2.8158	4.1945
ΔRSD	101	-9.8873*	48.908*	-9.9629*	33.111*	49.644*
RDC	102	-1.8420	1.6966	-1.8487	1.1802	1.7688
ΔRDC	101	-10.755*	57.858*	-10.703*	38.215*	57.318*
R9010	102	-4.2355*	8.9906*	-4.5131*	6.8119*	10.201*
ΔR9010	101	-11.167*	62.349*	-11.111*	41.160*	61.726*
R90C	102	-2.6822	3.6009	-2.7375	2.6313	3.9422
ΔR90C	101	-10.789*	58.196*	-10.782*	38.750*	58.119*
R10C	102	-1.5535	1.2136	-1.6759	1.3390	2.0035
ΔR10C	101	-9.2265*	42.570*	-9.2868*	28.755*	43.130*

* indicates significant at 10% level
RSC are the residuals from the regression logSPI = $\alpha_0 + \alpha_1$ logCPI
RSD are the residuals from the regression logSPI = $\alpha_0 + \alpha_1$ logSDiv
RDC are the residuals from the regression logSDiv = $\alpha_0 + \alpha_1$ logCPI
R9010 are the residuals from the regression BB90 = $\alpha_0 + \alpha_1$ TB10
R90C are the residuals from the regression BB90 = $\alpha_0 + \alpha_1$ logCPI
R10C are the residuals from the regression TB10 = $\alpha_0 + \alpha_1$ logCPI

Table 8. Error-Correction Model for 10 year Treasury Bond and 90 day Bank Bill yields

$$\Delta(TB10)_t = \beta_1\Delta(BB90)_t - (1-\alpha)((TB10)_{t-1} - \gamma_1 - \gamma_2(BB90)_{t-1})) + u_t$$

Coefficient	Parameter estimate	Standard Error	Prob >|T|
β_1	0.2325	0.0272	0.0001
$-(1-\alpha)$	-0.1123	0.0382	0.0041
γ_1	4.1960	0.4216	0.0001
γ_2	0.5905	0.0361	0.0001